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EFFECTS OF PIAGETIAN FORMAL OPERATIONS ON MATHEMATICS PERFORMANCE OF SENIOR SECONDARY STUDENTS IN KADUNA STATE, NIGERIA

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ABSTRACT: *Effect of formal operation abilities on mathematics performance of Senior Secondary Students (SSS) in Kaduna State, Nigeria was investigated. The population of the study was all the SSS III students in the state. Their number was 62,565. A sample of 400 students of equal numbers of males and females was drawn, from those offering arts and science subjects. Two paper and pencil tests were developed and used to generate data for analysis, namely, Formal Operations Test (FOT) and Mathematics Performance Test (MPT).The FOT was to assess students' level of formal operations while the MPT was developed to test their performance in mathematics. The results of the study showed that mathematics performance was low. Second, the numbers of students who always use and those who never use formal operations abilities were about equal. Third, results in the FOT and MPT were significant and positively correlated. Fourth, there was significant difference between the MPT scores of those students who always use and those who never use the abilities in favour of the former. Implications deduced were that this study confirmed that not all SS students use the abilities always and some never use them. Results of the analysis also indicated that formal operations significantly affect students' performance in mathematics. Those who always use the abilities performed better than those who never use them. This indicated that the widely experienced poor performance in mathematics, especially among senior secondary students, is partly as a result of students not always using formal operations abilities. Lastly, findings indicated that knowledge of students' level of formal operations can be used to understand, predict and improve their performance in mathematics. It was recommended that teachers of mathematics should endeavour to diagnose and identify their students' level of formal operations so that they help raise those students who sometimes use and those who never use the abilities to be using them always. It was also recommended that governments should be convening and sponsoring workshops that will evolve programs for accelerating students' cognitive development for use by teachers in normal mathematics lessons*.

ABSTRCT: Piagetian, Mathematics Performance, Senior Secondary Students, Kaduna, Nigeria

INTRODUCTION

Mathematics is one of the most important school subjects. This is because of the fact that mathematics finds applications in all school subjects and in the everyday lives of individuals.

It is one of the few subjects that are compulsory for all students from nursery to secondary school. Also, admission to read any science or related courses in Nigerian tertiary institutions is based on a requirement of a pass at credit level in O/level mathematics, among others. Despite this importance, Azuka (2000), Ibrahim (2003) and Kurumeh & Iji (2009) reported that performance in it has been constantly very poor over the years. Ibrahim (2003) reported that the percentage of Nigerian students who passed mathematics at credit level in West African Examinations Council for Senior Secondary Certificate (W.A.E.C/S.S.C.E) for the years 1981 to 1991 was averagely 14.4. Azuka (2000) also reported that students who passed mathematics at credit level in WAEC/SSCE for the years 1994 to 1996, in this country, was averagely 14.2. Again, Kurumeh & Iji (2009) reported that those who passed the subject at credit level in the same examinations for the year 2000 to 2006, in this country, was 24.4. This trend is really lamentable and pathetic, for it showed that on the whole, in the about twenty years in question, averagely only 4% of the students who sat for W.A.E.C examinations in this country were eligible for admission into the nation's tertiary institutions to study mathematics and physical and applied sciences. These are the courses every nation needs to advance technologically.

Several things could be said to be responsible for the present state of affairs in mathematics education. Reasons ranging from the nature of the subject itself, the students' attitudes, mathematics teachers and their methods and approaches in teaching the subject have been fingered as responsible (Azuka, 2000; Ezeugo & Agwagah, 2000; Ibe-Enwo, 2003; Ibrahim, 2003; Adekoya, 2009; Etukudo, 2009; Kurumeh & Iji, 2009 and Oyetunde & Emunefe, 2009). This study looked into whether a consideration of the patterns of thinking and reasoning common to Senior Secondary students, as proposed in Piaget's (1958) theory and stages of cognitive development, could help improve students' mathematics performance.

Background to the Study

No doubt, Jean Piaget is the father of cognitive psychology. He pioneered the school of cognitive psychology in his attempt to understand and explain human intellectual development. However, cognitive psychologists were not totally in agreement in their quest. Bee (1989) posited that the attempt to explain cognitive development turned out three different approaches of intelligence. The first is the individual differences approach that is based on the fact every individual differ in intellectual skills compared to others. Proponents of this approach ask how well a person does intellectual tasks compared to others. The second is the cognitive structure approach that is based, instead of on individual differences, on the patterns of reasoning and thinking common to individuals. Proponents of this approach emphasize that cognitive development is a result of the development of cognitive structures (schemata) which are, according to Piaget (Inhelder & Piaget, 1958), basic units of knowledge used to organize past experiences and serve as a basis for understanding new ones. They emphasize understanding the type of structure an individual uses in performing tasks and how these structures change with age. The third is information processing approach. Proponents of this approach based their work on understanding the basic processes that make up all intellectual activity. They emphasize understanding the basic processes an individual uses when faced with tasks, how the processes change with age and how individuals differ in their speed and skill in using them.

The cognitive structure approach, pioneered by Piaget, forms the spring board for this study. To him, cognitive development in humans could best be understood and explained not through a consideration of individual differences but through understanding the patterns of thinking common to them. This way, he discovered that children of similar ages have similar thinking and reasoning patterns and that they use these similar abilities when dealing with intellectual tasks. As a result of this discovery Piaget (1958) proposed his bold theory and stages of cognitive development.

Piaget, in his theory, proposed that every normal individual is born with built-in cognitive structures (schemata) that the individual use to interact with and explore the environment. This structure changes in unique and predictable patterns as the individual gains experience and learns new knowledge in the process of growing up. The change that occurs, in the cognitive structures, is generally similar with individuals of the same age group. From the theory of cognitive development, Piaget (1958) further proposed that individuals, from birth to adolescence, undergo cognitive transition through four stages. According to him, these stages are successively linked to each other, but are qualitatively different. At each stage children spontaneously acquire certain cognitive abilities that are peculiar to only children of that age group. For example every normal 3 to 5 year old will prefer that you give him five one naira notes instead of one five naira note. This is because children of this age group cannot conserve quantities and volumes. But, he observed that abilities acquired in one stage form the basis for transition to a higher succeeding stage.

Piaget cautioned that although children of any stage possess cognitive abilities and modes of thinking that are characteristic only to those children of that age group, the stages are such that all normal children pass through them in exactly the same order, without skipping. But the ages at which children progress through the stages may vary and the rate at which individual children pass through each stage may also differ. These inconsistencies, Piaget attributed to an individual's environmental setting, educational experiences, societal culture and natural ability. This implies that the abilities and stage acquired by individuals of the same age group may differ from society to society. The stages are the sensorimotor (from birth to 2years), the pre-operations (from 2 to 7 years), concrete operations (7 to 11 years) and formal operations (11 to 15 years).

The formal operations stage is the highest and last attained by individuals in the process of acquiring cognitive abilities that enables them to deal with their environment. Sweetland (2007) explained that attaining formal operations abilities means that for the first time in the adolescent's life he has the mental capacity to think as well as adults and has the ability to solve all types of problems. This implies that a formal operations individual is equipped to understand all the mathematics he is taught in the senior secondary school and is capable of solving all problems therein. Sweetland further explained that abilities acquired at the formal operations stage include combinations, proportionality, probability, hypothesis, correlation, metacognition, deductive logic, formal reasoning and isolation and controlling of variables. Incidentally, these abilities are mathematical in nature. All of them can be found as mathematics topics in the senior secondary syllabus or are used in solving mathematical problems. Piaget (1958) cautioned that there is what he called horizontal decalage at the formal operations stage. What this means is that out of those who have acquired the formal operations abilities; not all use them all the time and some never use them.

This study investigated whether the similarity between the structures of formal operations abilities and that of mathematics imply that the development and use of the formal operations abilities affect senior secondary students' performance in mathematics. Also, the study sought to find out whether a consideration of the thinking and reasoning abilities common to SS students, instead of their individual differences, will help improve their mathematics performance. It was hypothesized that SS students' performance in mathematics correlated with their level of formal operations and that SS students who always use the formal operations abilities performed better, in mathematics, than those who never use the abilities.

Two paper- and - pencil tests were developed and used to generate data in this study. The Mathematics Performance Test (MPT) was used to measure SS students' mathematics performance. The Formal Operations Test (FOT) was used to measure SS students' level of formal operations. Three similar studies created a vacuum that the present study has filled. Roberge & Flexer (1983) carried out a similar study on primary six and J.S1 and 2 students. Lawson, (1982) also carried out a similar study on J.S 3 students and Mwamwenda (1993) carried out same study on university first year students. The findings of the three sets of researches showed that students' level of formal operations significantly affect their mathematics performance.

METHOD

The population for this study was all the SS 3 students in Kaduna State. Their figure was 62,565, spread over 695 schools in the state. A sample of 400 students was used in the study. This figure was made up of 200 males and 200 females, drawn equally from those offering arts and science. The determination of the sample size of 200 for this study was informed by the suggestion advanced by McClave and Dietrich (1985). Ex post-facto research design was adopted in which two paper - and - pencil tests were used to generate data for the study. FOT and MPT were the tests used as instruments of data collection.

FOT

Roberge & Flexer (1983) in a similar study developed and used the Formal Operations Reasoning Test (FORT, 1983) to measure the level of formal operations of students. The FORT covered only three abilities of combinations, proportionality and deductive logic, while FOT was developed and used in this study to measure the same thing as the FORT but it covered five formal operations abilities of combinations, proportionality, deductive logic, probability and formal reasoning. The FOT contained 50 items made up of 10 items on each of the five abilities. Each of the items was scored out of 2 marks and the whole test out of 100 marks. Students' results on the test were used in determining their level of formal operations and as data for analysis.

Eight university lecturers, from the rank of lecturer II and above, drawn from the areas of mathematics, mathematics education and psychology validated the FOT. A pilot study was conducted in which the split-half method was used to obtain a linear correlation coefficient of +0.97 and a reliability coefficient of +0.99 at $p \leq 0.05$, using the Spearman – Brown formula. This established that the FOT actually measure what it was intended for and is highly consistent. Students' results on the FOT were used to classify them into three levels:

A = {those who always use formal operations abilities},
B = {those who sometimes use formal operations abilities} and
C = {those who never use formal operations abilities}.

The highest level was considered to be A, followed by B, then C. The mean score on the FOT ($\bar{x}$) and the standard error of the mean on the FOT (α) was used to define the three classifications of the use of the formal operations abilities.

$A = \{x : x > \bar{x} + \alpha\}; B = \{x : \bar{x} - \alpha \leq x \leq \bar{x} + \alpha\}; C = \{x : x < \bar{x} - \alpha\}$ *and* $\alpha = s/\sqrt{n}$.

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In this research, only those students whose use of formal operations abilities was clear cut were considered. The results of those who sometimes use the abilities were not considered in the analysis.

MPT

Algebra, geometry, trigonometry, number and numeration and everyday statistics are integral parts of modern mathematics. They also form the greater part of the Nigerian SS mathematics syllabus. The MPT was developed on these five major areas of SS mathematics, with one item from each. The items were scored out of 20 marks and the whole MPT out of 100. This test was used to measure SS students' mathematics performance. Students' results in this test were used as data in the analysis. Out of the eight lecturers that validated the FOT, six of them also validated the MPT. These were the ones drawn from the areas of mathematics and mathematics education. Two parallel tests of the MPT were used in a pilot study. Results showed a linear correlation coefficient of +0.76 and a reliability coefficient of +0.86 at $p \leq 0.05$, using the Spearman - Brown formula for reliability coefficient. This established that the test items measured what they were intended and that the test was highly consistent.

FOT Results

Students' results on the FOT showed that $\bar{x} = 39.03$, s = 22.71 *and* $\alpha = 1.14$. According to the defined levels of formal operations, namely A, B and C.

$$A = \{x : x > \bar{x} + \alpha\} = \{x : x > 39.03 + 1.14\}$$

$$= \{x: x > 40.17 \cong \{x: x > 40\}$$

$$B = \{x : \bar{x} - \alpha \leq x \leq \bar{x} + \alpha\}$$

$$= \{x: 39.03 - 1.14 \leq x \leq 39.03 + 1.14\}$$

$$= \{x: 37.89 \leq x \leq 40.17\} \cong \{x: 38 \leq x \leq 40\}$$

$$C = \{x : x < \bar{x} - \alpha\} = \{x : x < 39.03 - 1.14\}$$

$$= \{x: x < 37.89\} \cong \{x: x < 38\}$$

Thus, results on the FOT showed that number of students in level A, which was defined as those whose FOT score was more than 40 marks, was 200 or 50%.The next was level B, defined to be those whose FOT score was between 38 and 40 marks, their number was 18 or 4.5%. Lastly, level C, defined to be those whose FOT score was less than 38 marks, their number was 182 or 45.5% (Table 1).

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Table 1: Distribution of FOT results with the three levels of formal operations

Marks	No. of Students	Cumulative frequency	Level of formal operations	No. in level of formal operations
0 – 9	43	43		
10 – 19	72	115		
20 – 29	28	143		
30 – 37	39	182	**C** :never use	182 (45.5%)
38 – 40	18	200	**B** :sometimes use	18 (4.5%)
41 – 49	47	247		
50 – 59	49	296		
60 – 69	72	368		
70 – 79	24	392		
80 – 89	8	400		
90 – 100	0	400	**A** :always use	200 (50%)

MPT Results

Students' results on the MPT showed that $\bar{x}$ = 24.02 and s = 13.28. For level A, those students who always use formal operations abilities, $\bar{x}$ = 33.84 and s = 16.39. For level C, those who never use formal operations abilities, $\bar{x}$ = 11.72 and s = 10.16.
The PPMC and t-test statistics were used to test the two hypotheses in this study, respectively.
H_{01}:There is no significant relationship between the FOT and MPT scores.

Table 2: Summary of PPMC for FOT and MPT scores.

variables	n	$\bar{x}$	s	Df	r_{cal}	r_{crit}	p-value	decision
FOT	400	39.03	22.71	798	0.96	0.195	0.00	Significant
MPT	400	24.02	13.28					

The hypothesis was tested using the Pearson Product Moment Correlation technique. An r-value of +0.96 was found. The value was found to be significant at 0.05 level with df = 798. Hence, H_{01} was rejected in favour of its alternative.
H_{02}: The mathematics performance of SS students who always use and those who never use formal operations abilities are not significantly different.

Table 3: Summary of t-test analysis for level of formal operations abilities on SS students' MPT scores.

variables	n	$\bar{x}$	s	df	t_{cal}	t_{crit}	p-value	decision
always use	200	33.84	6.39	380	32.27	1.97	0.00	Significant
never use	182	11.72	10.16					

The t-test for independent samples was used. From analysis, the t-value (t_{cal} = 32.27) indicated significant difference and thus, H_{02} was rejected in favour of its alternative.

DISCUSSION

The results of students on the FOT and MPT have revealed a number of things. Students' performance was poorer in MPT than in FOT. Also, results of students in the FOT have shown that, among SS students, no particular level of formal operations dominates, between those who always use and those who never use formal operations abilities. This has confirmed Piaget's (1958) assertion that not all students who have developed the abilities use them always. Results of PPMC analysis on students' FOT and MPT scores indicated a high positive linear correlation coefficient between the two tests; implying that SS students' FOT scores can be used to predict their mathematics performance and students' mathematics performance can be used to identify their level of formal operations. In the same vein, results of the t-test analysis showed significant difference between the MPT scores of those students who always use and those who never use formal operations abilities, which indicated that SS students who always use the abilities performed better in mathematics than those who never use them. It follows, therefore, that the development and use of formal operations abilities affect SS students' mathematics performance. This means that the level of students' formal operations affect their performance in mathematics. This implies that coaching students to always use the abilities will help improve their performance in mathematics. Finally, this study has brought to light the importance of using the thinking and reasoning patterns common to students instead of their individual differences to understand, predict and improve their performance in mathematics.

Recommendations

The findings of this research have brought to light the importance of Piaget's theory and stages of cognitive development to mathematics education, generally, and to mathematics performance in the Senior Secondary, in particular. In this light, therefore, the following recommendations were made:

1. Mathematics teachers should endeavour to diagnose and identify, in their classes, students' level of formal operations so that they can use their lessons to train them to always use the abilities.
2. Government should be organizing and sponsoring workshops on the production of programs for accelerating the cognitive development of students through mathematics education.

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